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PROBABILITY OF NEGATION FOR CRUISE MISSILES USING LEAST DEFENDABLE ROUTES (U)

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Abstract

Probability of Negation P_N of an enemy missile depends upon its *path* from its launch point to its intended asset (target). Since Ballistic Missile (BM) *trajectories* can be predicted uniquely, once the BM's trajectory is known, then its P_N can be calculated in terms of the probabilities of success in the *three major functions*: Sensor, BM/C⁴I and Weapon. In contrast, the Cruise Missile (CM) *route* between its launch point and its intended asset is preplanned by the enemy, based upon his perception of the defense's performance and beddown, so that his CM will take the route of maximum *Probability of Survival* P_S (corresponding to minimum predicted P_N) while in transit. This particular route is called the *Least Defendable Route* (LDR). In our method, *Poisson density* is used to define a *risk field* (risk per unit route–length along source–type eight cardinal directions) in terms of *Probability of Detection, Engagement Volumes* (volumes of space where engagements are feasible) and *Engagement Lengths* (length between successive engagements for each engagement unit). The LDR between two points is found by directly maximizing P_S through minimizing the *cumulative risk* defined as the sum of risk along a route connecting those two points using the *D'Esopo–Pape Algorithm*. The resulting maximum P_S contour map represents the *offense's* perception of vulnerability. For the same LDR's, one can perform a model simulation, including additional details, and generate the *defense's* minimum P_N contour map. These two maps (P_S and P_N) provide *complementary* views for CM Defense.

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INTRODUCTION (U)

TAMD CRD produced by USACOM outlines the requirements of a Family of System (FoS) in terms of the Probability of Negation (P_N) of a threat. **ORIGIN**

P_N is one of the measures used to gauge the performance of a *Blue Force Deployment* against a *Threat* for the purpose of Acquiring the FoS's needed in a theater. **PURPOSE**

Accordingly, we want to calculate P_N using a method that is

- Fast
- Accurate
- Flexible

OBJECTIVE

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The Theater Air and Missile Defense *Capstone Requirements Document* (01 August 1998, JROCM 078–98) produced by the United States Atlantic Command outlines the requirements of a Family of System (FoS) in terms of the Probability of Negation of a threat. This is the ***origin*** of the definition of Probability of Negation.

Probability of Negation, or simply P_N, is one of the measures used to evaluate the performance of a Blue Force Deployment (Beddown) in a theater against an enemy threat for the ***purpose*** of *acquiring the FoS's* needed in that theater, in order to investigate whether they provide the required defense coverage stated in the CRD.

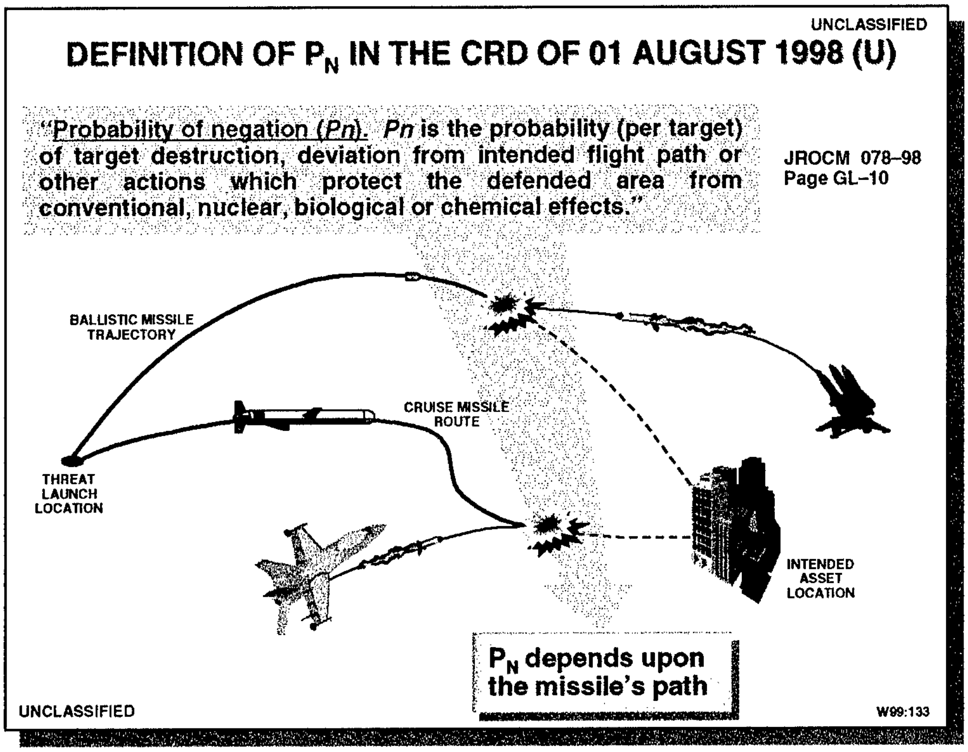
Accordingly, the ***objective*** is to calculate P_N using a method that is *Fast, Accurate and Flexible*. That is, different P_N methods must be compared in terms of the following three characteristics:

(1) ***Speed***—Computation of P_N must be fast enough so that performance of different Blue Force Beddowns can be compared within a reasonable time.

(2) ***Accuracy***—The method must provide P_N with sufficient accuracy so that reliable decisions can be made regarding FoS acquisition.

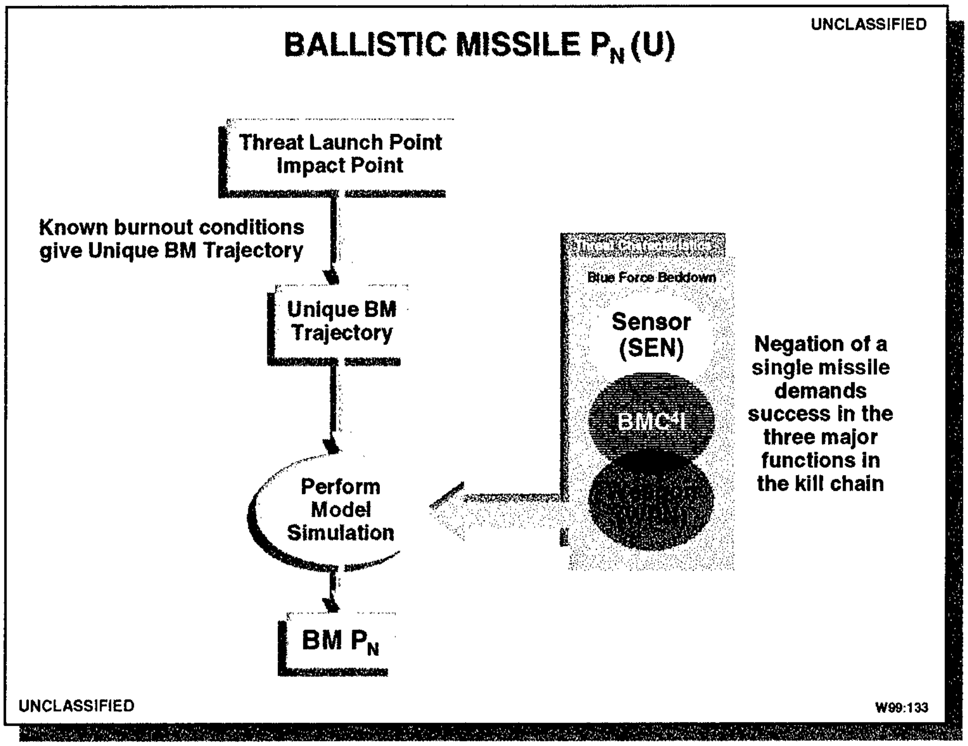
(3) ***Flexibility***—It must be easily extended to incorporate new features in the defense architecture.

These three points must be kept in mind.



This is the definition of P_N in the CRD (01 August 1998, JROCM 078–98, page GL–10). In this definition, "*target*" is interpreted as an enemy missile, either Ballistic or Cruise (one may include Aircraft as well, however, the calculation of its P_N is a difficult problem). Given the Beddown, the negation of a single missile is closely related to the flight path of that missile. We state this as follows: ***P_N depends upon the missile's path***.

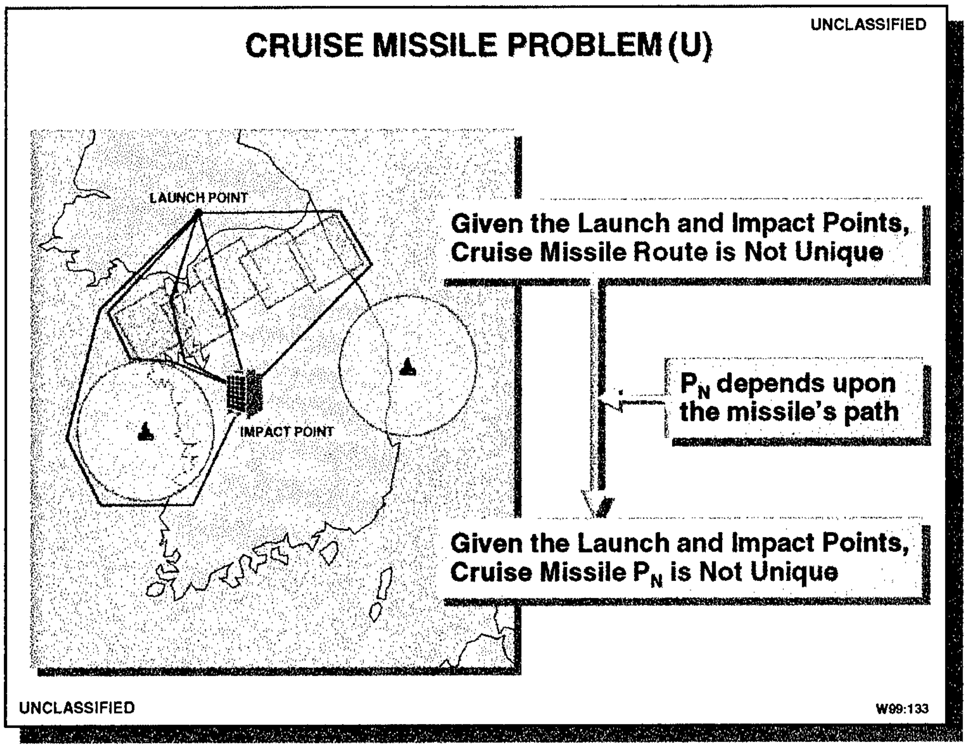
We reserve the term *trajectory* for Ballistic Missile's path and *route* for Cruise Missile's path.



Once the burnout conditions of a Ballistic Missile have become available, its trajectory, including its launch location and the intended impact point, can be predicted *fairly* uniquely. Then, simulating the Ballistic Missile flight along this trajectory in a model, one can calculate its P_N. The model simulation incorporates the dynamic or time varying nature of the kill chain, which consists of the following three major functions:

(1) Sensor – Detecting the missile,
(2) BM/C^4I – Deciding on how to intercept the missile,
(3) Weapon – Intercepting the missile.

In general, the Probability of Negation of a single Ballistic Missile, therefore, can be expressed schematically in terms of the probabilities of success in these three major functions.



In contrast, given a pair of launch and impact points, the Cruise Missile route is *not* unique, because it is preplanned by the enemy depending upon his intelligence about the Blue Force Beddown. Since P_N of a missile depends upon the path it takes, unless the Cruise Missile route is known, one cannot calculate the P_N using the knowledge of the launch and impact points alone.

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WHY MINIMUM P_N? (U)

How do we *standardize* the CM route and find a unique P_N?

Parallel to BM P_N Concept

Most Conservative Estimate

Highlights Defense Vulnerabilities

Concept of Minimum P_N and the route associated with it

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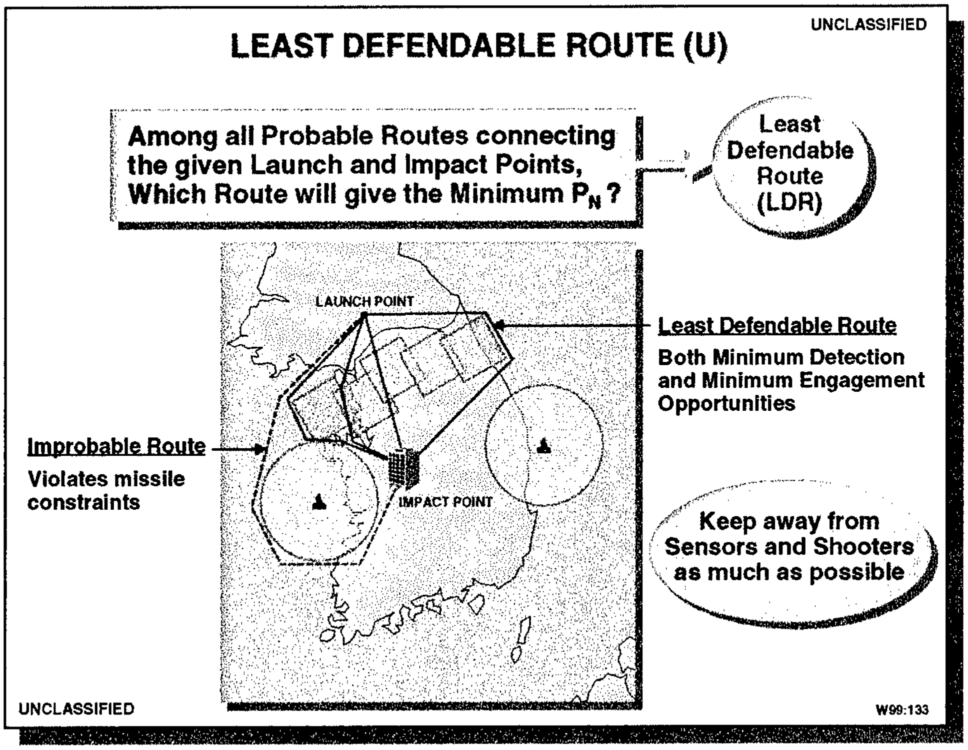
Given the launch and impact points, P_N for a BM is unique since its trajectory is uniquely determined by gravity. In other words, we can say that ***gravity standardizes the BM trajectory***. Then one could ask how do we standardize the CM route in order to get a unique P_N? This requires a *novel* concept to take the place of gravity. Since one may favor a CM P_N method:

- That is parallel to BM P_N concept,
- That gives most conservative estimate,
- That highlights defense vulnerabilities,

the concept of ***minimum P_N*** is introduced as a way to standardize the CM route.

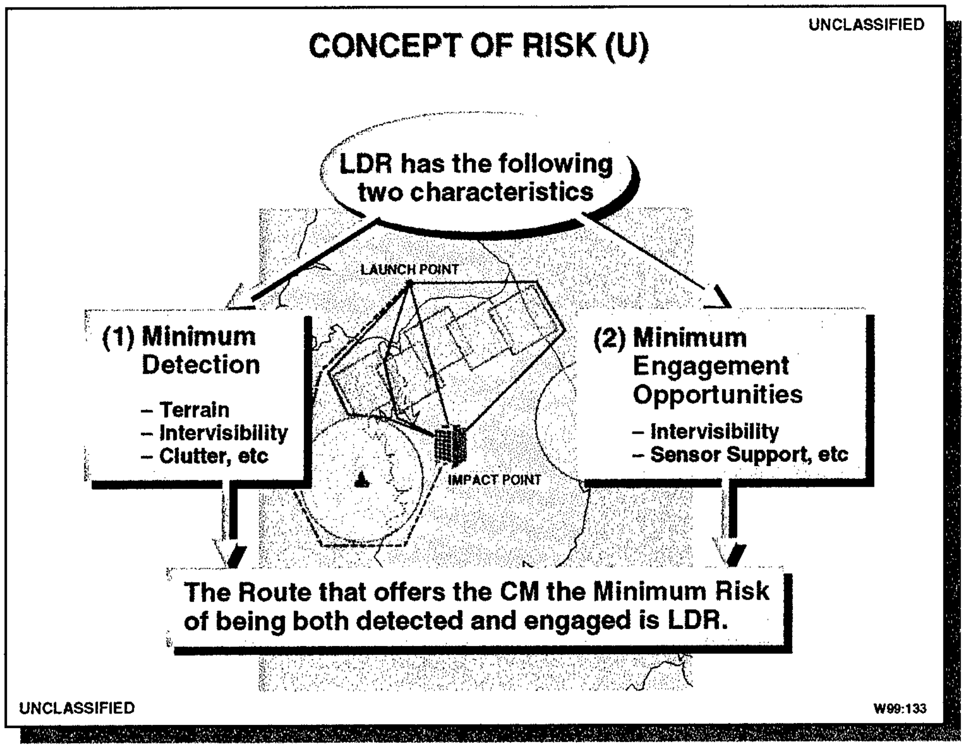
The <u>defense</u> wants to find out the CM route that is associated with the minimum Probability of Negation P_N while the <u>offense</u> wants to find out the route that has the maximum ***Probability of Survival*** P_S for its CM. Accordingly, these are *complementary* ideas.



Among the large number of routes connecting the given launch and impact points of a CM, some cannot be taken by the missile because of its design; for example, the missile can make only a limited number of turns, the g–limit restricts the missile from taking any sharp turns, compromise has to be made between missile maneuvering and route length (range) demanded by the limited amount of fuel onboard the missile, etc. These are referred to as the ***Improbable Routes*** while the rest are called the ***Probable Routes***.

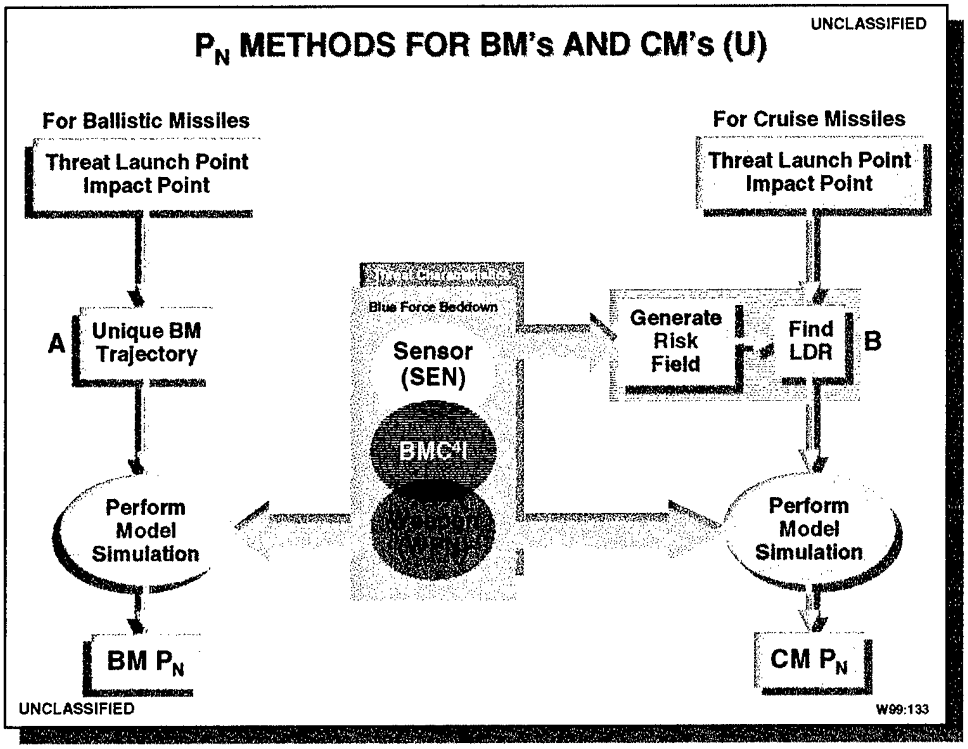
The question then is, *among all Probable Routes connecting the given pair of launch and impact points, which route will give the minimum P_N?* This particular route of minimum P_N is the ***Least Defendable Route*** (***LDR***) for the Blue Forces—Which threat (which Cruise Missile Route) worries the defense the most?

Successful negation of a missile involves both successful detection and engagement. The minimum P_N, therefore, results from both minimum detection and minimum engagement opportunities. Accordingly, the LDR is the route that has both minimum detection as well as minimum engagement opportunities. Simply, ***the route that keeps away from sensors and shooters as much as possible is the LDR***.



As noted earlier, the LDR possesses the following two properties: Minimum Detection and Minimum Engagement Opportunities. Factors such as terrain, intervisibility, clutter, sensor support, etc. are included in these.

The route that offers the CM the *Minimum Combined Risk of detection with engagement* is therefore the LDR. Thus the concept of a ***Risk Field*** is introduced. For the given *Blue Force Beddown* and the *Threat Characteristics* one must define an appropriate Risk Field which incorporates detection and engagement opportunities, then among all probable routes connecting the launch and impact points, the LDR is the one that has the ***Least Cumulative Risk***.



This exhibits the *parallel* between the P_N methods for BM's and CM's. The *only* difference is between boxes A and B. The three major functions in the kill chain, viz.,

> Sensor (intervisibility, clutter),
> BM/C⁴I (who can fire from whose sensor data),
> Weapon (P_K, sensor support, interceptor's speed and range),

as well as the Threat Characteristics (CM's altitude, speed, radar cross section) are used to construct a ***Risk Field***. Then the LDR between a given threat launch point and an impact point is the one that minimizes the cumulative risk along the route connecting these two end points. Finally, in a ***Model Simulation***, the CM is flown along this LDR and the P_N associated with it is calculated, thus incorporating the dynamic and the sequential nature of the kill chain in a global sense (i.e., for the entire LDR).

In the next few Slides, we introduce a simple derivation for equations involved in generating the Risk Field and the LDR's.

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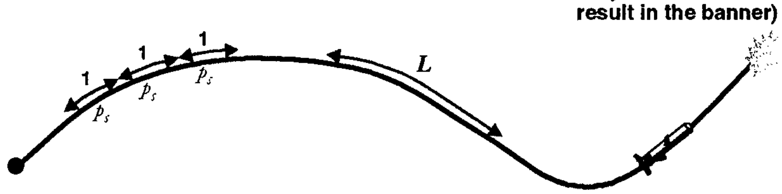
PROBABILITY OF SURVIVAL P_S (U)

Probability of Survival within unit route length $\equiv p_s$ = Constant everywhere

(This is assumed only to derive the result in the banner)



Probability of Survival within route length of 2 units $= p_s \times p_s = p_s^2$

Probability of Survival within route length of 3 units $= p_s \times p_s \times p_s = p_s^3$

Then by induction,

$$\left.\begin{array}{l}\text{Probability of Survival within}\\ \text{route length of } L \text{ units}\end{array}\right\} = p_s^{\,L}$$

This can be derived rigorously using the Poisson Distribution

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Explained in the Slide.

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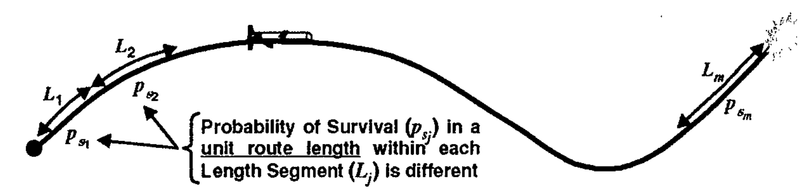
P_S ALONG THE ENTIRE ROUTE (U)



Probability of Survival of the CM along the *entire* route

$$P_S = p_{s_1}^{L_1} \times p_{s_2}^{L_2} \times \cdots \times p_{s_m}^{L_m}$$

$$\ln P_S = L_1 \ln p_{s_1} + L_2 \ln p_{s_2} + \cdots + L_m \ln p_{s_m}$$

$$\boxed{J \equiv -\ln P_S = \sum_{j=1}^{m} -\ln p_{s_j} L_j}$$

To next slide

$$\boxed{\int (-\ln p_s)\, dL}$$

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Explained in the Slide.

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RISK FUNCTION *R* (U)

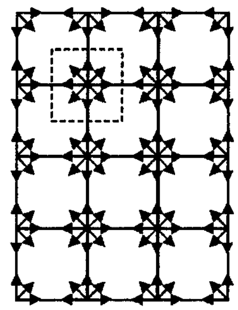
From previous slide

$$R_j \equiv -\ln p_{s_j}$$

Risk

$$R_j \equiv -\ln\left(1 - p_d\, p_{ek}\right)_j$$

$$J \equiv -\ln P_S = \sum_{j=1}^{m} -\ln p_{s_j} L_j$$



Probability of Detection (in unit length)	p_d (Location, 8 Directions)
Given Detection, the Conditional Probability of Engagement and Kill (in unit length)	p_{ek}(Location)
Local Probability of Negation (in unit length)	$p_n \equiv p_d \times p_{ek}$
Local Probability of Survival (in unit length)	$p_s \equiv 1 - p_n$

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Explained in the Slide.

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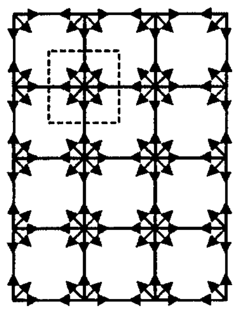
CUMULATIVE RISK *J* (U)

Risk

$$R_j \equiv -\ln p_{s_j} \qquad R_j \equiv -\ln\left(1 - p_d\, p_{ek}\right)_j$$

$$J \equiv -\ln P_S = \sum_{j=1}^{m} -\ln p_{s_j}\, L_j \qquad J = \sum_{j=1}^{m} R_j L_j$$

Cumulative Risk



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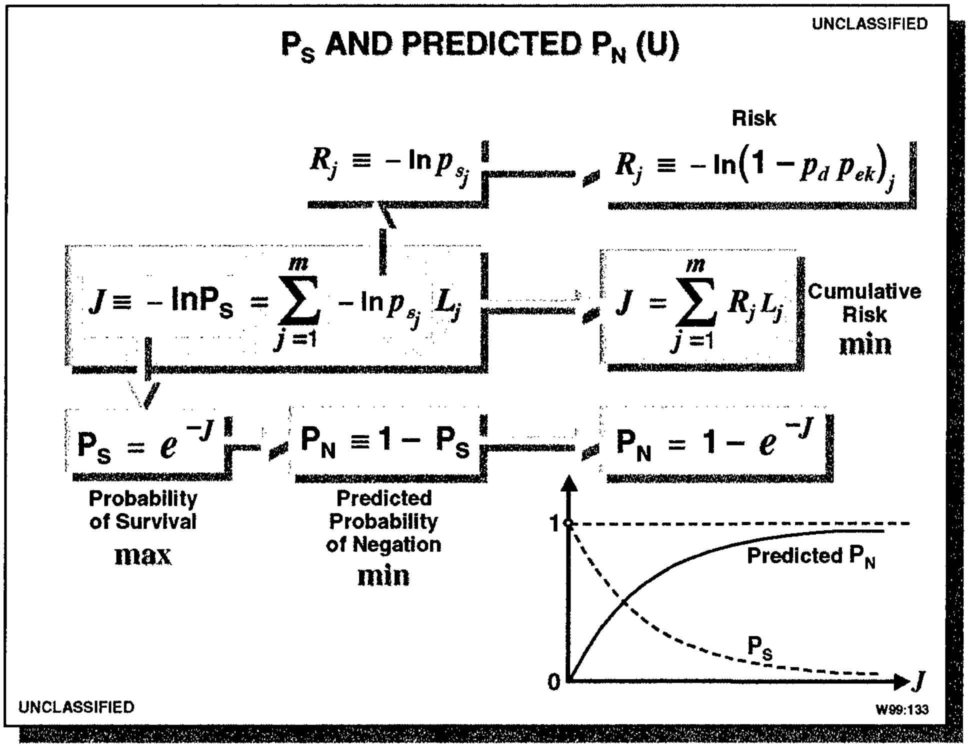
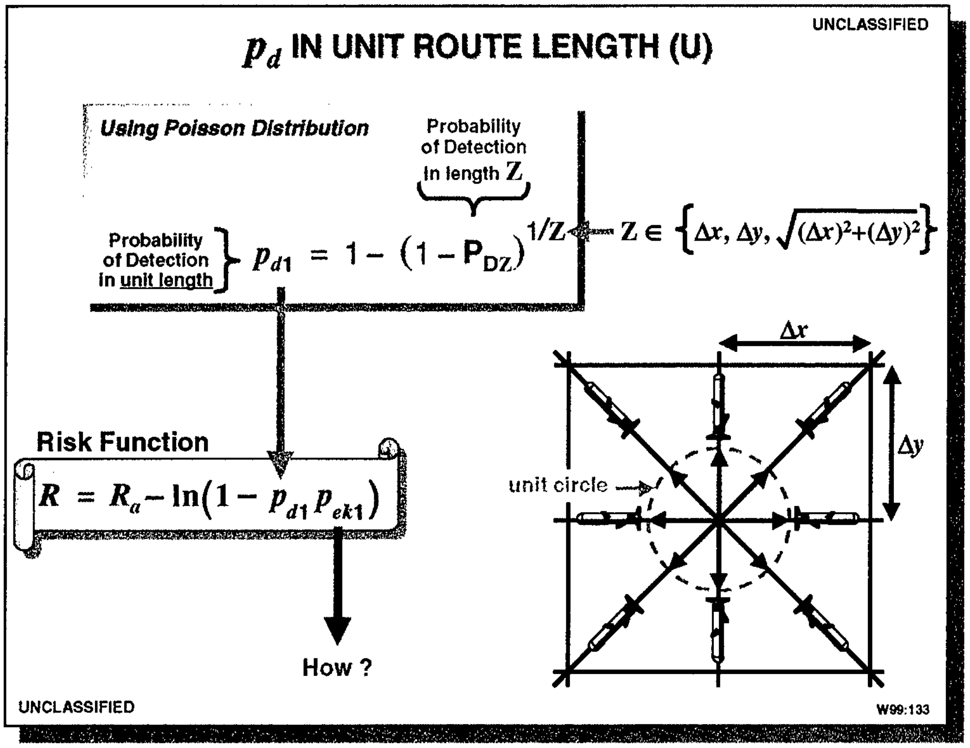


The offense wants the maximum P_S which corresponds to the defense's minimum Predicted P_N, and according to the graph on the right–hand bottom corner in this Slide, this occurs for the minimum cumulative risk *J*.

Since the ***Predicted P_N*** resulting from the *Probability of Survival P_S* does *not* incorporate the dynamic and the sequential nature of the kill chain in a *global sense*, it ***is not exactly the Probability of Negation P_N***. This Predicted P_N, however, can be used as a quick way to look at the defense vulnerabilities. The actual Probability of Negation P_N must be calculated in a model simulation using the LDR's.



By flying the CM at the specified altitude and speed along each cardinal direction, from a node to the adjacent node of distance Z apart along that cardinal direction, the probability of detection of the CM P_{DZ} in <u>length Z</u> is calculated for each cardinal direction at every node. Then, the probability of detection p_{d1} in a <u>unit length</u> is calculated using the formula shown in the box at the top of this Slide. Thus we have an eight–vector p_{d1} at every node.

In order to calculate the probability of engagement and kill p_{ek1} in a unit length, we introduce the concept of the ***Engagement Length*** Λ in the next Slide.

Also note that the formula for R (inside the banner) includes a nontrivial very small positive ***ambient risk*** R_a in order to make the portions of an LDR as straight as possible in regions where the risk is zero.

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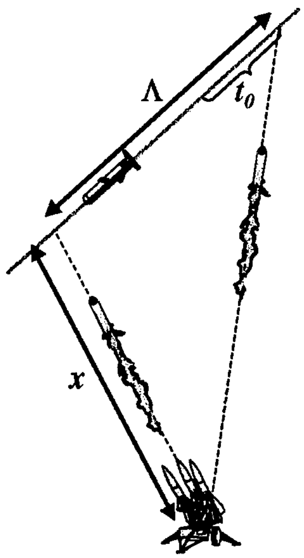
CONCEPT OF ENGAGEMENT LENGTH Λ (U)



Engagement Length Λ is defined as the distance between successive engagements

$$\left.\begin{array}{c}\text{Time between}\\ \text{successive}\\ \text{engagements}\end{array}\right\} = \frac{\Lambda}{V_{cru}} = \frac{x}{V_{int}} + t_0$$

$$\Lambda(x) = \frac{V_{cru}}{V_{int}}x + V_{cru}t_0$$

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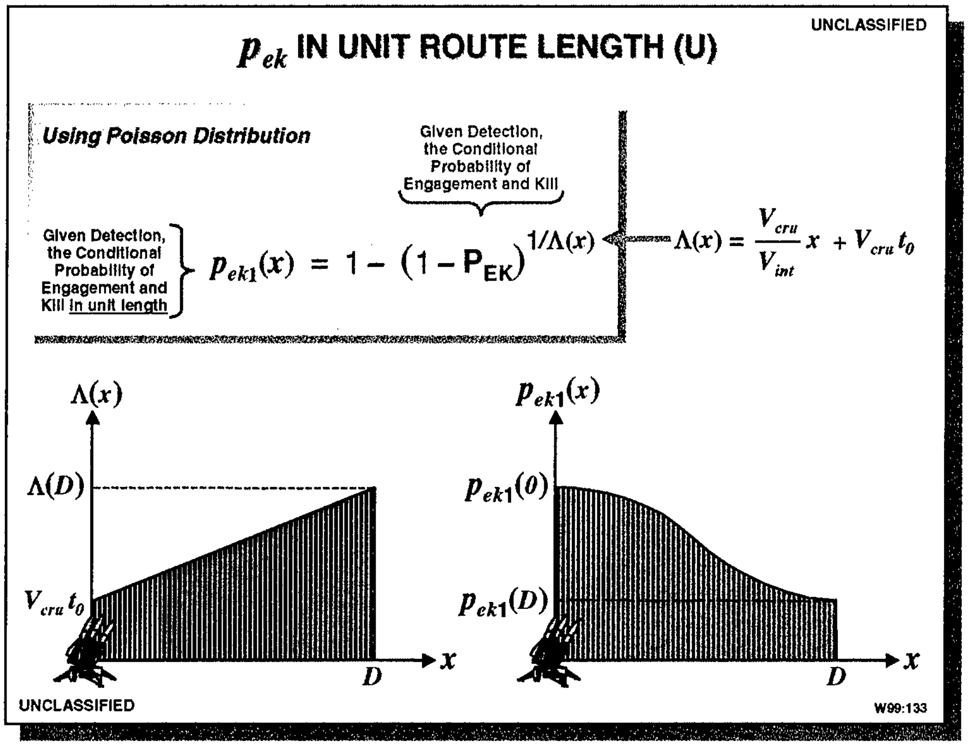
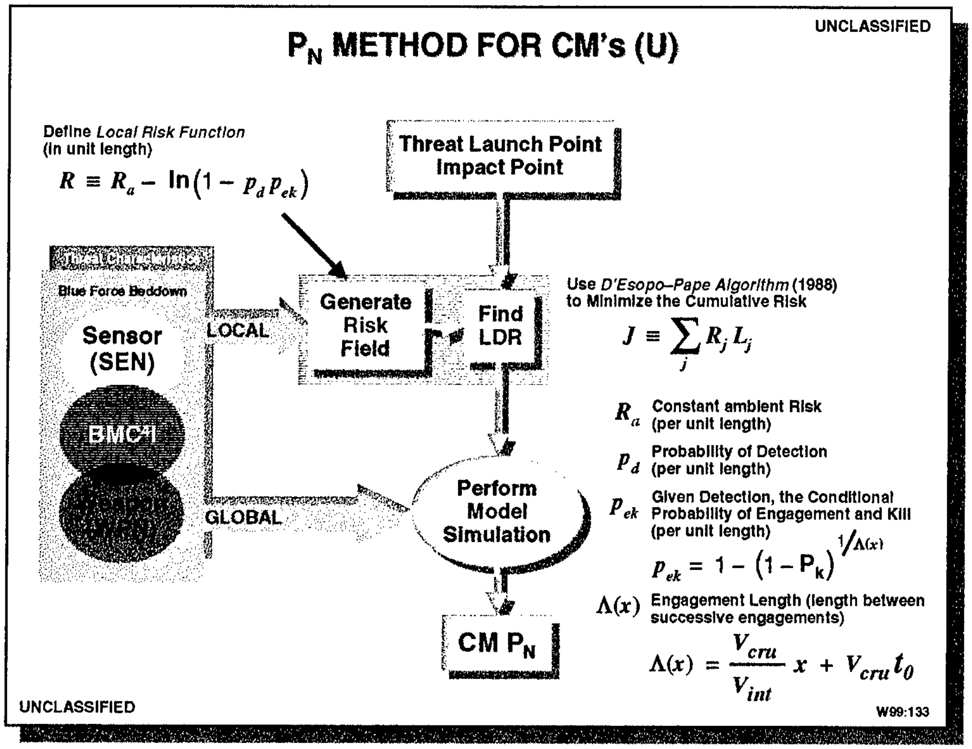
This defines the Engagement length Λ and gives a simple formula to calculate it. Here t_0 is a time delay (e.g., kill assessment time).



The probability of engagement and kill P_{EK} of a ***single shot*** is transformed into the probability of engagement and kill p_{ek1} in a unit length using the formula in the box at the top of this Slide. Even if one assumes a constant P_{EK}, the p_{ek1} in unit length varies with distance x from the engaging unit as shown by the graph on the right–hand bottom corner. Here D is the maximum interceptor range supported by a fire control sensor.

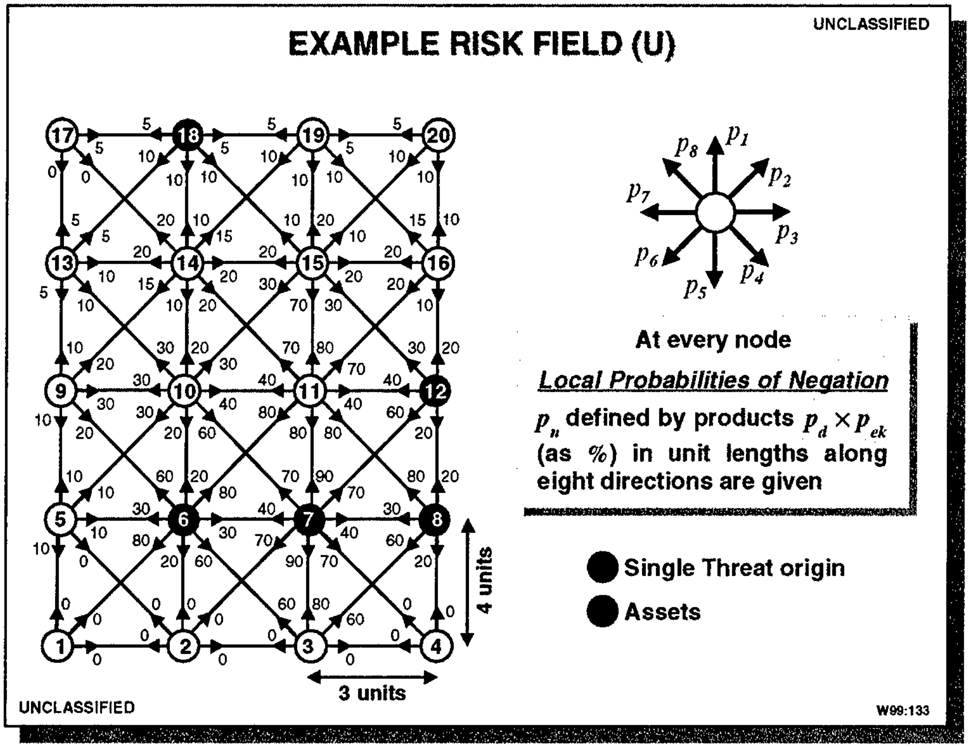


Although in the last few Slides we gave a simple derivation of the ***Risk Function R*** (shown at the left–hand top corner of this Slide) and other related formulas, they can be derived *rigorously* using the *Poisson Distribution.* The Risk Function R describes the Risk Field locally.

In a regular grid, at every grid point (node) the risk per unit route length R is given along *eight "cardinal "directions* in terms of the probability of detection p_d in unit route lengths along eight cardinal directions, probability of engagement and kill p_{ek} in a unit route length, and also a very small nontrivial positive, isotropic and homogeneous ambient risk R_a that is introduced to make the route unique in an area where the risk is zero. The p_{ek} is obtained in terms of the Probability of Kill P_K and the length Λ traveled by the CM between successive engagements, that is expressed as a function of the interceptor speed V_{int}, the CM speed V_{cru}, the kill assessment time t_0 and also the intercept location x from the interceptor.

Given the Risk Field R, the ***Cumulative Risk*** J is defined as the sum of the product $R_j \times L_j$, where R_j is the risk in a unit route length between successive waypoints (nodes) of distance L_j apart. The ***D'Esopo–Pape (1988) Shortest Path Finding Algorithm*** is used to minimize J in order to find the LDR; i.e., the waypoints along the LDR. (Here, the *Shortest Path* should not be interpreted literally as shortest distance, rather as the least risk).

We demonstrate this method using a simple example in the next Slide.



The Example describes a battlefield consisting of a single threat origin (launch point, node 18) and four different assets (nodes 6, 7, 8 and 12). Drawing the diagonals in each grid box, every node at one of the four outer corners is connected to three of its adjacent (immediate neighbor) nodes while a node in the boundary is connected to five adjacent nodes and an interior node is connected to eight adjacent nodes.

The product $p_d \times p_{ek}$ in the Risk Function $\boldsymbol{R}$ can be viewed as the *Local Probability of Negation* p_n *in a unit route length.* The values of this product in eight ''cardinal'' directions (as %) are given as shown.

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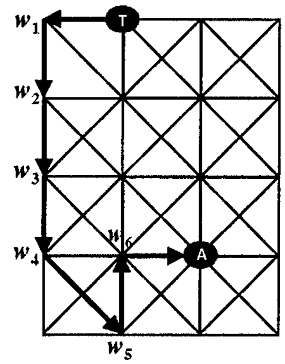
ALGORITHMS TO FIND THE LDR's (U)

For a given (T, A) pair, find the Node Sequence

$$\mathbb{W} \equiv \{T, w_1, w_2, \ldots, w_M, A\},$$

which is a subset of all the nodes, such that

$$J[\mathbb{W}] \equiv \sum_{w_j \in \mathbb{W}} R(w_j)L(w_j) \quad \text{is the minimum}$$



A Minimal Spanning Tree Algorithm can be used to find the LDR

Bellman–Ford Algorithm
Dijkstra Algorithm
D'Esopo–Pape Algorithm

These Algorithms do not handle any Constraints. We include the Missile Range Constraint by modifying the Risk Field using a Lagrange Multiplier later.

We use the D'Esopo–Pape (1988) Algorithm to find the LDR's from a single threat node to all other nodes.

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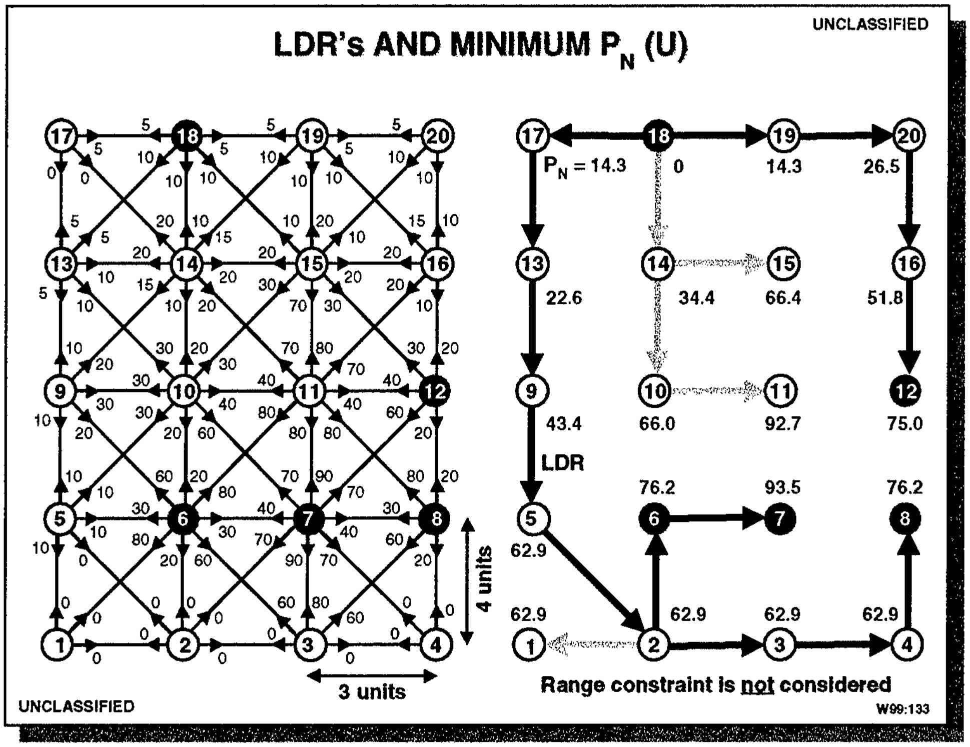
Given the threat origin T and the intended impact point (asset) A, the aim is to find the ***Node Sequence*** $\mathbb{W}$, whose first element is T and the last element is A, such that the cumulative risk J is the minimum. In other words, the problem is to find the *intermediate waypoints* $w_1, w_2, \ldots, w_M$ in this Node Sequence $\mathbb{W}$. Accordingly, the number of intermediate waypoints M is an unknown too.

This problem falls under the class of finding the ***Shortest Path or the Minimal Spanning Tree*** described in the areas of Network Optimization, Graphs and Networks, and Dynamic Programming. Several Algorithms such as *Bellman–Ford* (1958), *Dijkstra* (1959) and *D'Esopo–Pape* (1988) are developed to find the shortest paths from a single node to all other nodes in a network. We use the ***D'Esopo–Pape Algorithm*** to establish the LDR's from a single threat origin to all the assets.

These Algorithms do not handle any constraints. We, however, include the ***Missile Range Constraint*** by modifying the Risk Field using a ***Lagrange Multiplier***.



The picture on the right–hand side shows the LDR's to the four assets in black and the LDR's to other nodes in gray. The ***Predicted Minimum*** P_N defined by

$$P_N \equiv 1-e^{-J},$$

where J is the minimum cumulative risk, is displayed in red adjacent to each node. The defense's P_N contour map, however, has to be calculated in a model simulation using the same LDR's.

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CAVEATS ON SAMPLE RESULTS (U)

In order to demonstrate our method, the following simplifying inputs were used only in the risk field generation. These simplifications, however, are not required by our risk function.

- Engagement envelopes for surface–based engagement units were assumed to be circles
- Engagement envelopes for FEZ's were modeled by rectangles
- $P_K = 0.7$ was assumed for each system (including aircraft)

A Beddown in a Notional Scenario was used in this demonstration.

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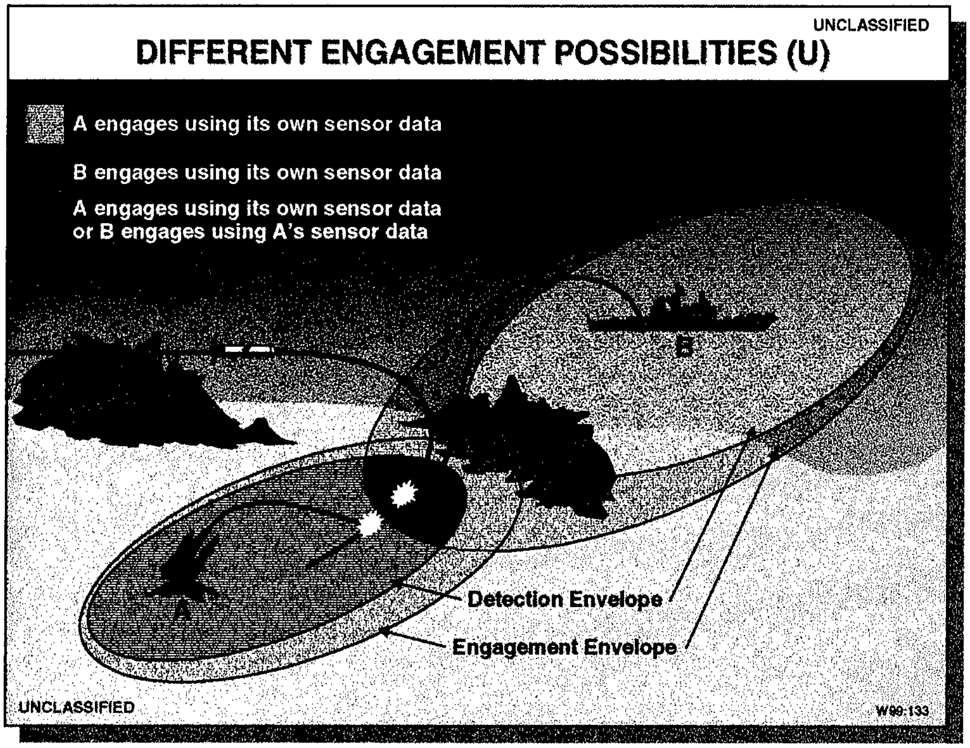
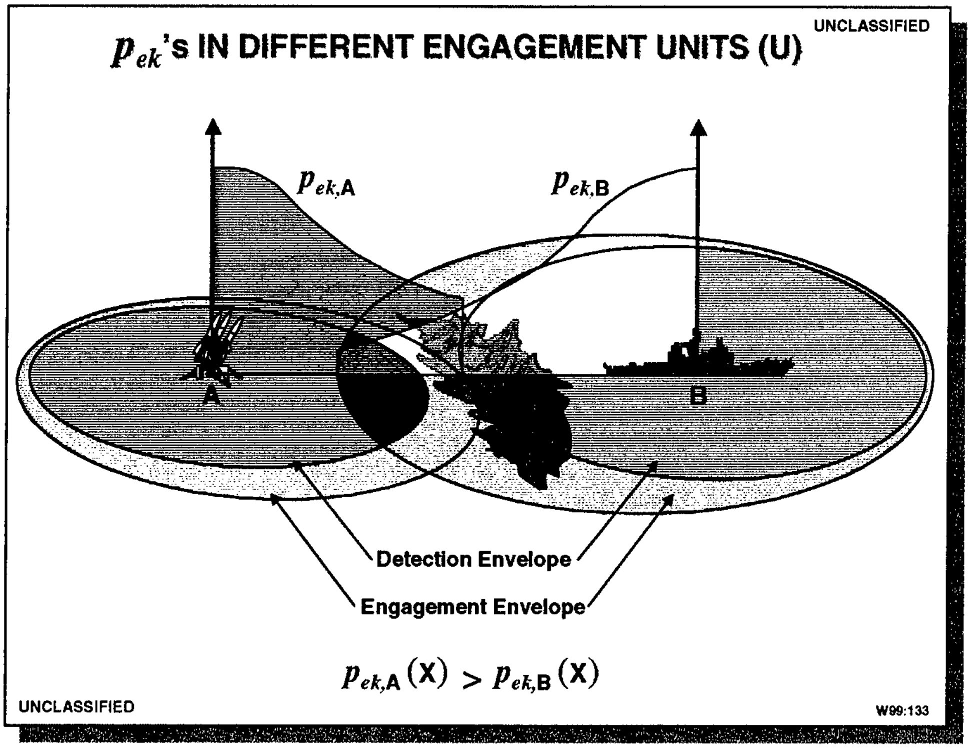


Explained in the Slide.



Explained in the Slide.

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RANGE CONSTRAINT (U)

For a given (T, A) pair, find the Node Sequence

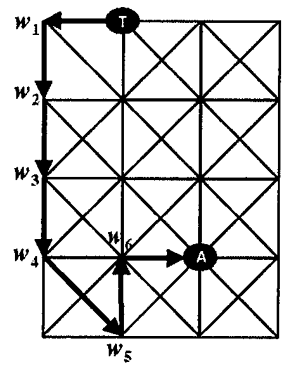
$$\mathrm{W} \equiv \{\mathrm{T}, w_1, w_2, \ldots, w_M, \mathrm{A}\},$$

which is a subset of all the nodes, such that

$$J[\mathrm{W}] \equiv \sum_{w_j \in \mathrm{W}} R(w_j) L(w_j) \quad \text{is the minimum}$$

subject to the *Range Constraint*

$$K[\mathrm{W}] \equiv \sum_{w_j \in \mathrm{W}} L(w_j) \leq \text{Missile Range } M_R$$



Find W and the Lagrange Multiplier λ minimizing

$$I[\mathrm{W}] \equiv J[\mathrm{W}] + \lambda K[\mathrm{W}]$$

such that

$$K[\mathrm{W}] \leq M_R$$

Risk Function

$$R = R_a - \ln(1 - p_{d1} p_{ek1})$$
$$\tilde{R} = R + \lambda$$

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Explained in the Slide.